Time-Reversal Space Division Multiple Access over Multi-path Channels

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Abstract

A novel concept of time-reversal division multiple access (TRDMA) is proposed in [1] as a low-complexity, energy-efficient wireless multi-user access method. In this paper, we first generalize the work in [1] as a pre-filtering based space division multiple access (SDMA) communication system, where each user's signature waveform is derived from the information of CIRs between basestation (BS) and location-specific multiple mobile stations (MS). We develop two pre-filtering schemes: one is based on the scheme as proposed in [1], we refer it as time-reversal matched filter (TRMF). The benefit of the TRMF is to fully collect energy at the sampling instant of the intended MS (spatial and temporal focusing). The second pre-filter, which is referred to as zero-forcing pre-filter (ZFPF), is derived to meet the zero-forcing criterion such that multiuser interference (MUI) is completely eliminated. However, since the uplink CIR is generally different from the downlink CIR, we present extensive analysis on the performance degradation of the ZFPF caused by this mismatched effect. Furthermore, to improve the robustness to mismatch, we propose a simple yet effective robust zero-forcing pre-filter (R-ZFPF). We investigate and compare the performances of both schemes under various scenarios in terms of power consumption and averaged signal-to-interference-plus-noise power ratio (SINR). It is shown in both analytical and simulation results that satisfying performance can be achieved by the proposed pre-filtering system. Moreover, we demonstrate that the proposed R-ZFPF comprehensively improves the performance in mismatch condition.

Key words*: *Space Division Multiple Access (SDMA), Time-reversal Matched-filter (TRMF), Zero-forcing Pre-filter (ZFPF), Multiuser interference (MUI).

1. Introduction

The time-reversal (TR) transmission technique has been well-studied in wireless communication systems [2] for its full use of multi-path propagation and low-complexity of channel estimation. It has also been applied in underwater acoustic channel [3], ultra-wideband (UWB) communication system [4-7]. Recently, it has been shown that TR signal transmission is an ideal paradigm for green wireless communications because of its inherent nature to fully harvest energy from the surrounding environment [8]. A general TR communication system involves two stages: channel estimation and data transmission stages. In the first stage, each end user emits a delta-like pilot pulse that propagates to the fixed access point or basestation (BS) through a richly scattering medium. Then the BS records the multi-path channel impulse response (CIR) that is unique for each user as long as their physical locations are different. Data transmission occurs in the second stage, where BS utilizes the location-specific spatial signatures to remove multiuser interference (MUI) to the intended MS. Attractive features of TR signal transmission include:

(1) It makes full use of the energy from surrounding environment by exploiting the multi-path propagation: it can create space and time focalization at a specific point where signals are coherently added.
(2) Channel estimation in dense multi-path environment is generally a difficult task. The TR technique shifts the sophisticated channel estimation burden from the receivers of MS to the BS. This is also referred to as the "pre-rake" diversity combining scheme [9].
(3) Timing acquisition (synchronization) in TR scheme is extremely simplified since the peak is automatically created and aligned of the received signal at specific time slot.
(4) In a rich scattering wireless environment, the multi-path reflection profile between the BS and intended MS can be regarded as a location-specific spatial signature that is unique for the intended MS. Toward this end, a time-reversal division multiplexing (TRDMA) scheme is proposed in multi-path channels [1] for a multi-user downlink communication system.

In the first part of this paper, we construct a general pre-filtering based multi-user downlink system over multi-path channels. In the proposed structure, a bank of pre-filters are designated for each user at the transmitter of BS. The outputs are combined and then transmitted by single antenna (though extension to antenna array is without conceptual difficulty). Two types of pre-filters are developed that fully utilize the characteristics of the location-specific signatures between the BS and multiple MSs to suppress MUI. Similar to the scheme proposed in [1, 10], the impulse response (IR) of the first type pre-filters is the time-reversed conjugation of the CIR. Therefore, the multi-path channel serves as a matched filter (MF) to the transmitted waveform, and we refer it as the TRMF scheme. Since the TRMF has excellent spatial-temporal focusing capability, the energy of the received signal tends to concentrate on specific time instants. Hence, a peak is automatically formed at the desired MS and at the end of every bit. This enables us to implement a simple MS receiver to extract the energy at these sampling instants where peak occur. The IR of the second type pre-filters is derived to meet the zero-forcing (ZF) criterion such that MUI is completely removed at the sampling instants of the intended MS receiver. We refer it as the ZFPF scheme. The rationale of the pre-filtering based space division multiple access (SDMA) system is similar to the direct sequence code division multiple access (DS/CDMA) scheme. The pseudorandom (PN) sequence in DS/CDMA corresponds to the IR of the pre-filter. Nevertheless, the receiver of DS/CDMA system, which includes synchronization, multi-user detection [11], and channel estimation, is much more complicated than the proposed scheme. Specifically, since the signature waveform is time-varying and adapted according to CIR, the information is not possible to be decrypted by the eavesdropper. However, in conventional DS/CDMA system, the information is easily intercepted as long as the generating algorithm of PN sequence is known by the eavesdropper.

Most past works of TR signal transmission assume reciprocity between the uplink and downlink channels. Thus, the uplink CIR measured (estimated) at the first stage can be exploited at the second stage for signal transmission and multi-user separation. In practice, however, there indeed exists unavoidable discrepancy. We refer the estimation error in the first stage as “mismatch”. This paper aims to provide a systematic analysis for the performance (in terms of the averaged signal-to-interference-plus-noise power ratio (SINR)) degradation induced by mismatch effect. We deduce that the MS near the BS is more sensitive to mismatched effect, which is referred to the “Near-far effect” under mismatch scenario. To improve the robustness of ZFPF to the mismatch effect, we propose a robust ZFPF (R-ZFPF) scheme that preserves reliable performance in the presence of mismatch. Simulation results under different scenarios demonstrate that the SINR of the proposed R-ZFPF outperforms the system without robust processing to a large extent.

The remainder of this paper is organized as follows. In section 2, we introduce the multi-path channel model and formulate the pre-filtering based multi-user downlink signal model. Section 3 highlights the rationale of the proposed two pre-filtering schemes and investigates the performance under ideal case. The performance degradation due to mismatch between uplink and downlink channels is extensively analyzed in section 4. Moreover, we exploit power constraint on the zero-forcing criteria to develop robust pre-filters. Simulation results in terms of the averaged SINR are presented and analyzed in section 5. Concluding remarks are finally made in section 6.

Notation: The boldface letters represent vector or matrix. $\mathbf{A}(i, j)$ denotes the element of ith row and jth column of matrix $\mathbf{A}$. $\mathbf{x}(l)$ denotes the lth element of vector $\mathbf{x}$. $[\ ]^T, [\ ]^H$ stand for transpose and complex transpose of a matrix or vector, respectively. We will use $E\{\ \}$ for expectation (ensemble average), $\|\ \|$ for vector norm, and $\equiv$ for “is defined as”. “*” indicates the linear convolution operation. $\mathbf{I}_M$ denotes an identity matrix with size M. $\delta(\cdot)$ is the dirac delta function. $\overline{x}$ denotes the complex conjugate of x, $\hat{x}$ denotes the estimate of x.

2. Signal and channel models

2.1 Channel model

In this paper, we consider a multi-user downlink network over rich multi-path fading channel. The CIR of the communication link between the BS and k-th MS is modeled as

$$h_k(t) = \sum_{l=0}^{L} \alpha_{k,l}\delta(t - lT_c)\,;\ k=1,\ldots,K \tag{1}$$

where $\alpha_{k,l}$ is the gain of the l-th multi-path component (MPC) of the link between BS and k-th MS. To simplify the notation, we assume $\{\alpha_{k,l}\}_{\substack{k=1,\ldots,K\\ l=0,\ldots,L}}$ are real-valued, though extension to complex-value is without difficulty. T_c denotes the resolvable time. To simplify the analysis, we assume the channel parameters are quasi-static (slowly fading) such that they are essentially constant over the observation interval. Note that in writing (1), we have modeled the multi-path channel as a tapped-delay line with (L+1) taps. $\alpha_{k,l}$ denotes the tap weight of the l-th resolvable path. Moreover, we have implicitly assumed that maximum time dispersion (delay spread) is $T_d = LT_c$.

2.2 Signal model

We denote the sequence of information symbols (bits) for the *k*-th user by $\{d_k(i)\}$ and assume to be *i.i.d.* random variables, which takes on the value ± 1 with equal probability. Let T_b be the bit period (the duration by which consecutive bits $\{d_k(i)\}$ are separated), then the binary phase-shift-keying (BPSK) signal can be modeled as

$$s_k(t) = \sum_{i=-\infty}^{\infty} d_k(i)\,\delta(t - iT_b) \quad ; k=1,\ldots,K \tag{2}$$

In the proposed pre-filtering scheme, a set of *K* pre-filters with IR $\{g_k(t)\}_{k=1,\ldots,K}$ are inserted, respectively, between $\{s_k(t)\}_{k=1,\ldots,K}$ and the transmitting antenna in order to pre-equalize the multi-path fading channels and mitigate the MUI. A pre-filtering based downlink multi-user communication system is depicted in Fig. 1, where the BS and all MSs are equipped with a single antenna. The transmitted waveform, $x(t)$, at the BS is given by

$$x(t) = \sum_{j=1}^{K} s_j(t) * g_j(t) = \sum_{j=1}^{K} \sum_{i=-\infty}^{\infty} d_j(i)\, g_j(t - iT_b) \tag{3}$$

Based on (3), the energy consumption for the *k*-th user within one bit can be calculated as

$$E_{b,k} = \int_{T_b} g_k^2(t)\,dt \tag{4}$$

The received signal at the *k*-th MS can be formulated as

$$r_k(t) = x(t) * h_k(t) + n_k(t) \tag{5}$$

where $n_k(t)$ is assumed to be zero-mean AWGN noise process with variance σ_k^2. Please note that the bit duration, which is up to our disposal, is chosen to be equal to the delay spread (maximum dispersion of the CIR), $T_b = T_d = LT_c$. Substituting (1), (3) into (5), we have

$$\begin{aligned} r_k(t) &= \left(\sum_{j=1}^{K} s_j(t) * g_j(t) \right) * h_k(t) + n_k(t) \\ &= \sum_{j=1}^{K} s_j(t) * p_{jk}(t) + n_k(t) \\ &= \sum_{j=1}^{K} \sum_{i=-\infty}^{\infty} d_j(i)\, p_{jk}(t - iT_b) + n_k(t) \end{aligned} \tag{6}$$

where $p_{jk}(t) \triangleq g_j(t) * h_k(t)$. As shown in Fig. 1, at the front end of each mobile receiver, we sample every bit, sign test is then followed to determine the transmitted bit stream.

$$\hat{d}_k(i) = \mathrm{sgn}\left[r_k(iT_b) \right] \tag{7}$$

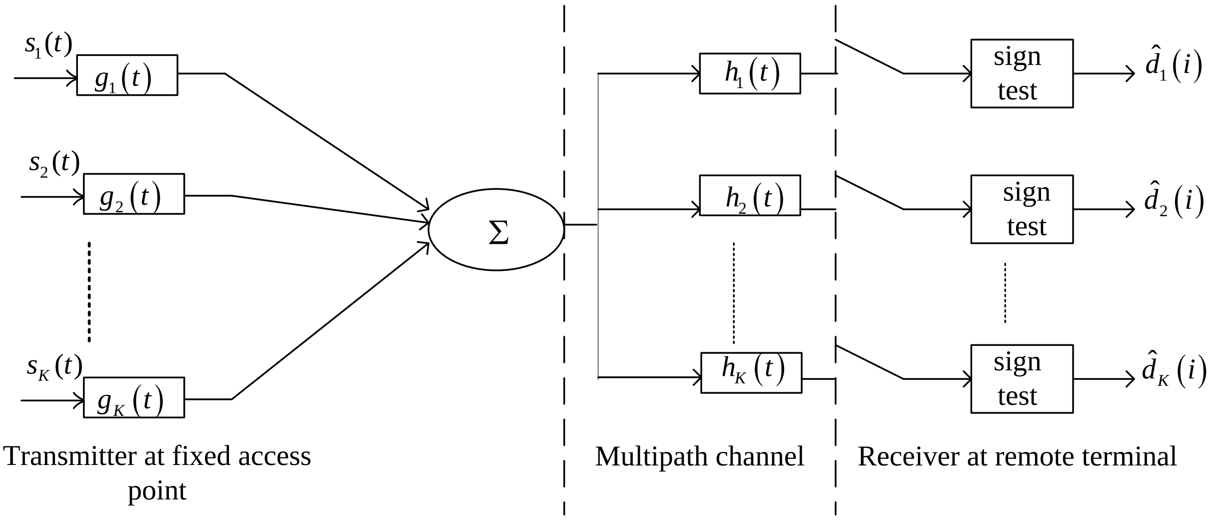


Fig. 1: Structure of a pre-filtering based downlink space division multiple access communication system

3. Design of pre-filtering based SDMA system

It is worthy to note that channel reciprocity (uplink and downlink share the same IR) is essential for applying the pre-filtering technique. If channel reciprocity is satisfied, we may estimate the downlink CIR by receiving sounding pulses (the sounding pulse should be made short enough to approach $\delta(t)$) from each of the MS. Therefore, the transmitter has full knowledge of the channel state information. Then, a set of pre-filters with IR $\left\{g_k(t)\right\}_{k=1,\ldots,K}$ are designed based on the estimated CIRs $\left\{h_k(t)\right\}_{k=1,\ldots,K}$. Two types of pre-filters are proposed in this paper. The first scheme, which is referred to as TRMF, utilizes the time-reversed CIR (with conjugation if $\left\{\alpha_{k,l}\right\}_{\substack{k=1,\ldots,K\\ l=0,\ldots,L}}$ are complex-valued) as their signaling waveforms. Therefore, the multi-path channel serves as a MF to the transmitted waveform. The rationale is equivalent to direct-sequence spread spectrum (DSSS) system, where pre-filter and the multi-path channel correspond to the spreader and de-spreader, respectively. The second scheme, which is referred to as ZFPF, fully utilizes the all the CIRs and the pre-filter is designed to meet the zero-forcing (ZF) criterion such that MUI is completely removed at the sampling instants of the intended MS receiver. It should be noted that the receiver at MS is extremely simple, due to the fact that the signal processing burden is shifted to BS.

To simplify the analysis, we assume the order of the FIR pre-filters being the same as CIR, though extension to any order is possible. Denoting the discrete-time version of the IRs of channel ($h_k(t)=\sum_{l=0}^{L}\alpha_{k,l}\delta\left(t-lT_c\right)$) and pre-filters ($g_k(t)=\sum_{l=0}^{L}\beta_{k,l}\delta\left(t-lT_c\right)$), as $(L+1)$ vectors $\mathbf{h}_k \equiv \left[\alpha_{k,0} \quad \alpha_{k,1} \quad \cdots \quad \alpha_{k,L}\right]^T$, $\mathbf{g}_k \equiv \left[\beta_{k,0} \quad \beta_{k,1} \quad \ldots \quad \beta_{k,L}\right]^T$, respectively, then the discrete-time counterparts of $p_{jk}(t)?\ g_j(t)\quad h_k(t)$;" $j,k = 1,\mathrm{K}\ ,K$ can be obtained as

$$\mathbf{p}_{jk}=\begin{bmatrix} \alpha_{k,0} & 0 & \cdots & \cdots & 0 \\ \vdots & \alpha_{k,0} & \ddots & \ddots & \vdots \\ \alpha_{k,L} & \vdots & \ddots & \ddots & \vdots \\ 0 & \alpha_{k,L} & \ddots & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & \alpha_{k,0} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \alpha_{k,L} \end{bmatrix}\begin{bmatrix} \beta_{j,0} \\ \beta_{j,1} \\ \vdots \\ \beta_{j,L} \end{bmatrix} \tag{8}$$

where $\mathbf{p}_{jk}$ is a vector with size $(2L+1)$. It is evident from (8) that

$$\mathbf{p}_{jk}\left(L+1\right)=\sum_{l=0}^{L}\alpha_{k,L-l}\beta_{j,l}=\tilde{\mathbf{h}}_k^T\mathbf{g}_j \tag{9}$$

where $\tilde{\mathbf{h}}_k \equiv \left[\alpha_{k,L} \quad \cdots \quad \alpha_{k,1} \quad \alpha_{k,0}\right]^T$. In what follows, at the sampling instant of k-th MS is

$$r_k\left(iT_b\right)= d_k\left(i\right)\tilde{\mathbf{h}}_k^T\mathbf{g}_k + \sum_{\substack{j=1\\ j\neq k}}^{K} d_j\left(i\right)\tilde{\mathbf{h}}_k^T\mathbf{g}_j + n_k\left(iT_b\right) \tag{10}$$

where the first term at the right-hand-side of (10) is the desired signal, the second term stands for the MUI. It follows that the averaged SINR can be obtained as

$$SINR_k=\frac{\left|\tilde{\mathbf{h}}_k^T\mathbf{g}_k\right|^2}{\sum_{\substack{j=1\\ j\neq k}}^{K}\left|\tilde{\mathbf{h}}_k^T\mathbf{g}_j\right|^2+\sigma_k^2} \tag{11}$$

3.1 TRMF scheme

In TRMF scheme, we utilize the time reversed version of CIR as the IR of pre-filter. Thereby, a set of TRMFs with IRs

$$g_k\left(t\right)=\eta\frac{h_k\left(T_d-t\right)}{\int_{T_b} h_k^2\left(t\right)dt}=\frac{\eta}{\sum_{l=0}^{L}\left|\alpha_{k,L-l}\right|^2}\sum_{l=0}^{L}\alpha_{k,L-l}\delta\left(t-lT_c\right);k=1,\ldots,K \tag{12}$$

are placed at the transmitter as the pre-filters, where η is a positive constant that accounts for the gain provided by the BS such that $g_k(t) * h_k(t) = \eta; \forall k$, *i.e.*, $\tilde{\mathbf{h}}_k^T\mathbf{g}_k=\eta$. It is worthy to note that in TRMF scheme, the time-reversed CIR is employed as the unique signature for each user and the multi-path channel forms a matched filter to the transmitted waveform $g_k\left(t\right)$. Rewrite (12) in vector form, we have

$$\mathbf{g}_{k,TRMF}=\frac{\eta}{\left\|\tilde{\mathbf{h}}_k\right\|^2}\tilde{\mathbf{h}}_k \tag{13}$$

Note that if the channel coefficients are complex numbers, we will use $g_k\left(t\right)=\eta\bar{h}_k\left(T_d-t\right)$ instead. It is easy to deduce that the energy consumption by k-th user of the TRMF scheme can be obtained as

$$P_{k,TRMF}=\left\|\mathbf{g}_k\right\|^2=\frac{\eta^2}{\left\|\tilde{\mathbf{h}}_k\right\|^2}=\frac{\eta^2}{\left\|\mathbf{h}_k\right\|^2};\ k=1,\ldots,K \tag{14}$$

Eqn. (14) reveals the fact that the power required for a target SINR is inversely proportional to $\left\|\mathbf{h}_k\right\|^2$. Smaller $\left\|\mathbf{h}_k\right\|^2$ accounts for large path loss which implies the distance between MS and BS is large. Hence, BS should enhance the transmit power to compensate the propagation conditions that the signal is experiencing at downlink communication.

Substituting (13) into (10), we arrive at

$$r_k\left(iT_b\right)=\eta d_k\left(i\right)+\eta\sum_{\substack{j=1\\ j\neq k}}^{K} d_j\left(i\right)\frac{\tilde{\mathbf{h}}_k^T\tilde{\mathbf{h}}_j}{\left\|\tilde{\mathbf{h}}_j\right\|^2}+n_k\left(iT_b\right) \tag{15}$$

where the second term stands for the MUI. The SINR of the TRMF scheme can be obtained as

$$SINR_{k,TRMF}=\frac{\eta^2}{\sum_{\substack{j=1\\ j\neq k}}^{K}\frac{\eta^2}{\left\|\mathbf{h}_j\right\|^4}\left|\tilde{\mathbf{h}}_k^T\tilde{\mathbf{h}}_j\right|^2+\sigma_k^2} \tag{16}$$

It follows that as the undesired users are far from BS, more power should be emitted to compensate path loss.

This leads to the increase of MUI, which severely degrades system performance. Therefore, the SINR of the MS near the BS is worse than the MS that is far from the BS. We refer the phenomena as the "near-far problem" in pre-filtering system.

3.2 ZFPF scheme

The ZFPF scheme aims to design pre-filters to completely remove MUI at the sampling instant of the desired MS. Define the composite IR of k-th pre-filter and j-th CIR as $p_{kj}(t) \equiv g_k(t) \otimes h_j(t)$, thus the k-th pre-filter should be designed to meet the following ZF criteria:

$$p_{kj}(LT_c) = \begin{cases} 0; & j \neq k \\ \eta; & j = k \end{cases} \qquad \forall j,k = 1,\ldots,K \tag{17}$$

where LT_c is the delay introduced to accommodate the multi-path effect. We may reformulate (17) as

$$\mathbf{p}_{kj}(L+1) = \sum_{l=0}^{L} \alpha_{j,L-l}\beta_{k,l} = \tilde{\mathbf{h}}_j^T \mathbf{g}_k = \begin{cases} 0; & j \neq k \\ \eta; & j = k \end{cases} \tag{18}$$

Upon defining the (L+1) by K matrix, $\tilde{\mathbf{H}} \equiv \left[\tilde{\mathbf{h}}_1 \quad \tilde{\mathbf{h}}_2 \quad \cdots \quad \tilde{\mathbf{h}}_K \right]$, (18) can be rewritten as a compact form

$$\tilde{\mathbf{H}}^T \mathbf{g}_k = \eta \mathbf{e}_k \tag{19}$$

where $\mathbf{e}_k$ denotes the k-th column vector of $\mathbf{I}_K$. If $K \leq (L+1)$, which is usually the case in dense multi-path environment, we have infinitely many solutions since (19) is indeed an underdetermined system. The minimum-norm solution can be obtained as

$$\mathbf{g}_{k,ZFPF} = \eta \tilde{\mathbf{H}} \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right]^{-1} \mathbf{e}_k \; ; k=1, \ldots, K \tag{20}$$

The energy consumption for k-th user of the ZFPF scheme can be obtained as

$$\begin{aligned} P_{k,ZFPF} &= \left\| \mathbf{g}_k \right\|^2 = \eta^2 \mathbf{e}_k^T \left(\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right)^{-1} \mathbf{e}_k \\ &= \eta^2 \left(\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right)^{-1} (k,k) \end{aligned} \tag{21}$$

Intuitively, the ZFPF scheme automatically adjusts the transmitted power for each user to match the propagation (attenuation) condition. That is, a reduction in signal level received at the BS will result in an increase in the radiated power from the BS. Moreover, it is also derived that the power required for ZFPF exceeds the power needed in TRMF, *i.e.*, .

$$P_{k,ZFPF} > P_{k,TRMF} \; ; k = 1, \ldots, K \tag{22}$$

Based on the zero-forcing criterion, the i-th sample at the k-th MS receiver is

$$r_k(iT_b) = \eta d_k(i) + n_k(iT_b) \tag{23}$$

As we compare (23) with (15), it is evident that the PZR scheme is free from MUI. The averaged SINR at the k-th MS yields

$$SINR_{k,ZFPF} = \frac{\eta^2}{\sigma_k^2} \tag{24}$$

Regardless of the possible difference of the background noise power, $\left\{ \sigma_k^2 \right\}_{k=1,\ldots,K}$, the SINR measured at each MS is essentially the same, which means there is no near-far problem in ZFPF scheme.

It is well-known of conventional DS/CDMA system, applying a ZF filter (or equivalently, decorrelating detector) in the MS receiver to remove MUI will enhance the additive background noise [11]. While the proposed ZFPF scheme is free from the noise enhancement problem. Moreover, to implement the decorrelating detector at MS requires the information of all the PN codes as well as all the CIRs, which in practical situation, has difficulty for the MS to acquire these information. On the other hand, it is not a problem for the ZFPF scheme since the required information is available at BS.

4. Mismatch effect and robust processing on the ZFPF scheme

4.1 Impact of mismatch effect on the ZFPF scheme

Both the TRMF and ZFPF schemes premise on perfect knowledge of the CIR between BS and each MS. Almost all the present works related to time-reversal processing assume reciprocity between the downlink and uplink channels and exploits the uplink CIR to model downlink CIR. However, in practical situation, there is unavoidable mismatch between the nominal and actual downlink CIR. To characterize the effect of mismatch on the SINR degradation, let $\left\{\mathbf{h}_i\right\}_{i=1,\ldots,K}$ and $\left\{\hat{\mathbf{h}}_i\right\}_{i=1,\ldots,K}$ denote the actual and nominal downlink CIR, respectively. Mismatch occurs whenever the BS assumes that spatial signatures are $\left\{\hat{\mathbf{h}}_i\right\}_{i=1,\ldots,K}$, whereas the true signatures are $\left\{\mathbf{h}_i\right\}_{i=1,\ldots,K}$. Let $\hat{\tilde{\mathbf{H}}} \equiv \begin{bmatrix} \tilde{\mathbf{h}}_1 & \tilde{\mathbf{h}}_2 & \cdots & \tilde{\mathbf{h}}_K \end{bmatrix}$, we should modify (19) as

$$\hat{\tilde{\mathbf{H}}}^T \mathbf{g}_k^{(mis)} = \eta \mathbf{e}_k \tag{25}$$

The minimum-norm solution can be obtained as

$$\mathbf{g}_{k,ZFPF}^{(mis)} = \eta \hat{\tilde{\mathbf{H}}} \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right]^{-1} \mathbf{e}_k \text{ ; } k=1,\ldots,K \tag{26}$$

Therefore, the averaged SINR measured at k-th MS becomes

$$\begin{aligned} SINR_{k,ZFPF}^{(mis)} &= \frac{\left| \tilde{\mathbf{h}}_k^T \mathbf{g}_{k,ZFPF}^{(mis)} \right|^2}{\sum_{\substack{j=1 \\ j\neq k}}^{K} \left| \tilde{\mathbf{h}}_k^T \mathbf{g}_{j,ZFPF}^{(mis)} \right|^2 + \sigma_k^2} \\ &= \frac{\left(\frac{\eta}{\sigma_k}\right)^2 \left| \tilde{\mathbf{h}}_k^T \hat{\tilde{\mathbf{H}}} \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right]^{-1} \mathbf{e}_k \right|^2}{\left(\frac{\eta}{\sigma_k}\right)^2 \sum_{\substack{j=1 \\ j\neq k}}^{K} \left| \tilde{\mathbf{h}}_k^T \hat{\tilde{\mathbf{H}}} \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right]^{-1} \mathbf{e}_j \right|^2 + 1} \\ &\xrightarrow{\left(\frac{\eta}{\sigma_k}\right)^2 >> 1} \frac{\left| \tilde{\mathbf{h}}_k^T \hat{\tilde{\mathbf{H}}} \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right]^{-1} \mathbf{e}_k \right|^2}{\sum_{\substack{j=1 \\ j\neq k}}^{K} \left| \tilde{\mathbf{h}}_k^T \hat{\tilde{\mathbf{H}}} \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}} \right]^{-1} \mathbf{e}_j \right|^2} \end{aligned} \tag{27}$$

Note that the performance of the TRMF scheme under mismatch condition can be deduced as

$$\begin{aligned} SINR_{k,TRMF}^{(mis)} &= \frac{\left| \tilde{\mathbf{h}}_k^T \mathbf{g}_{k,TRMF}^{(mis)} \right|^2}{\sum_{\substack{j=1 \\ j\neq k}}^{K} \left| \tilde{\mathbf{h}}_k^T \mathbf{g}_{j,TRMF}^{(mis)} \right|^2 + \sigma_k^2} \\ &= \frac{\left(\frac{\eta}{\sigma_k}\right)^2 \left| \frac{\tilde{\mathbf{h}}_k^T \hat{\tilde{\mathbf{h}}}_k}{\left\| \hat{\mathbf{h}}_k \right\|^2} \right|^2}{\left(\frac{\eta}{\sigma_k}\right)^2 \sum_{\substack{j=1 \\ j\neq k}}^{K} \left| \frac{\tilde{\mathbf{h}}_k^T \hat{\tilde{\mathbf{h}}}_j}{\left\| \hat{\mathbf{h}}_j \right\|^2} \right|^2 + 1} \xrightarrow{\left(\frac{\eta}{\sigma_k}\right)^2 >> 1} \frac{\left| \frac{\tilde{\mathbf{h}}_k^T \tilde{\mathbf{h}}_k}{\left\| \mathbf{h}_k \right\|^2} \right|^2}{\sum_{\substack{j=1 \\ j\neq k}}^{K} \left| \frac{\tilde{\mathbf{h}}_k^T \tilde{\mathbf{h}}_j}{\left\| \mathbf{h}_j \right\|^2} \right|^2} \end{aligned} \tag{28}$$

From the above discussion, we may draw the following remarks:

(1) In mismatched condition, the zero-forcing scheme loses MUI-removal capability. Moreover, we can observe from (27) that mismatched effect makes the SINR insensitive (asymptotically independent) to the variation

of $\left(\frac{\eta}{\sigma_k}\right)^2$.

(2) Near-far effect under mismatch scenario: It follows from (27) that the level of MUI is proportional to the magnitude (norm) of $\left\{\mathbf{h}_j\right\}_{j\neq k}$. From the fact that the transmission power assigned for each user is inversely proportional to $\left\|\mathbf{h}_k\right\|^2$, thus we can draw the conclusion that the MS near the BS is more sensitive to mismatched effect.

4.2 Robust processing

As described in the previous section, the ZFPF scheme will experience severe performance degradation in accordance with mismatch. We have also shown that the sensitivity of the ZFPF scheme is dominated by the transmission power. Hence, improved robustness to mismatch can be achieved by adding power constraint, or equivalently, setting quadratic (power) constraint on the design of the pre-filters. Intuitively, modify the ZFPF toward TRMF can make it more robust (less sensitive) to the mismatch effect. Hence, it is desirable to modify the zero-forcing criterion of (19) to the constrained optimization problem

$$\begin{cases} \arg\min_{\mathbf{g}_k} \mathbf{g}_k^T \hat{\tilde{\mathbf{R}}} \mathbf{g}_k \\ subject \quad to \begin{cases} \hat{\tilde{\mathbf{h}}}_k^T \mathbf{g}_k = \eta \\ \left\|\mathbf{g}_k\right\|^2 = \chi_k \end{cases} \end{cases} \tag{29}$$

where $\widehat{\tilde{\mathbf{R}}} \equiv \sum_{k=1}^{K} \tilde{\mathbf{h}}_k \tilde{\mathbf{h}}_k^T = \tilde{\mathbf{H}}\tilde{\mathbf{H}}^T$. χ_k is the additional (quadratic) constraint to limit the transmission power for the kth subscriber. And the value of χ_k should be set between the power needed for TRMF and ZFPF

$$\frac{\eta^2}{\left\|\mathbf{h}_k\right\|^2} \leq \chi_k \leq \eta^2 \left[\tilde{\mathbf{H}}^T \tilde{\mathbf{H}}\right]^{-1}(k,k) \tag{30}$$

Imposing the constraints of (29) by Lagrange multipliers, we may construct the cost function as

$$J(\mathbf{g}_k) = \mathbf{g}_k^T \hat{\tilde{\mathbf{R}}} \mathbf{g}_k + \delta_k\left(\mathbf{g}_k^T \mathbf{g}_k - \chi_k\right) - \lambda_k\left(\tilde{\mathbf{h}}_k^T \mathbf{g}_k - \eta\right);\ k=1,...,K \tag{31}$$

Equating the gradient of the cost function with respect to $\mathbf{g}_k$ to zero, we have

$$\hat{\tilde{\mathbf{R}}} \mathbf{g}_k + \delta_k \mathbf{g}_k = \lambda_k \tilde{\mathbf{h}}_k;\ k=1,...,K \tag{32}$$

or equivalently,

$$\mathbf{g}_{k,ZFPF}^{(rob)} = \lambda_k \left(\hat{\tilde{\mathbf{R}}} + \delta_k \mathbf{I}_{L+1}\right)^{-1} \tilde{\mathbf{h}}_k;\ k=1,...,K \tag{33}$$

where λ_k is chosen in order that the constraint $\hat{\tilde{\mathbf{h}}}_k^T \mathbf{g}_k = \eta$ is satisfied. Solving for λ_k gives

$$\mathbf{g}_{k,ZFPF}^{(rob)} = \frac{\eta\left(\hat{\tilde{\mathbf{R}}} + \delta_k \mathbf{I}_{L+1}\right)^{-1} \tilde{\mathbf{h}}_k}{\hat{\tilde{\mathbf{h}}}_k^T \left(\tilde{\mathbf{R}} + \delta_k \mathbf{I}_{L+1}\right)^{-1} \tilde{\mathbf{h}}_k};\ k=1,...,K \tag{34}$$

The resulting SINR can be obtained by substituting (34) into (11), which yields

$$SINR_{k,ZFPF}^{(rob)}=\frac{\left|\tilde{\mathbf{h}}_k^T\mathbf{g}_k^{(rob)}\right|^2}{\sum_{\substack{j=1\\k\neq j}}^{K}\left|\tilde{\mathbf{h}}_k^T\mathbf{g}_j^{(rob)}\right|^2+\sigma_k^2}=\frac{\left(\frac{\eta}{\sigma_k}\right)^2\left|\frac{\tilde{\mathbf{h}}_k^T\left(\hat{\tilde{\mathbf{R}}}+\delta_k\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_k}{\hat{\tilde{\mathbf{h}}}_k^T\left(\tilde{\mathbf{R}}+\delta_k\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_k}\right|^2}{\left(\frac{\eta}{\sigma_k}\right)^2\sum_{\substack{j=1\\k\neq j}}^{K}\left|\frac{\tilde{\mathbf{h}}_k^T\left(\hat{\tilde{\mathbf{R}}}+\delta_j\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_j}{\hat{\tilde{\mathbf{h}}}_j^T\left(\tilde{\mathbf{R}}+\delta_j\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_j}\right|^2+1}$$

$$\xrightarrow{\left(\frac{\eta}{\sigma_k}\right)^2>>1}\frac{\left|\frac{\tilde{\mathbf{h}}_k^T\left(\hat{\tilde{\mathbf{R}}}+\delta_k\mathbf{I}_{L+1}\right)^{-1}\hat{\tilde{\mathbf{h}}}_k}{\hat{\tilde{\mathbf{h}}}_k^T\left(\tilde{\mathbf{R}}+\delta_k\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_k}\right|^2}{\sum_{\substack{j=1\\j\neq k}}^{K}\left|\frac{\tilde{\mathbf{h}}_k^T\left(\hat{\tilde{\mathbf{R}}}+\delta_j\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_j}{\hat{\tilde{\mathbf{h}}}_j^T\left(\tilde{\mathbf{R}}+\delta_j\mathbf{I}_{L+1}\right)^{-1}\tilde{\mathbf{h}}_j}\right|^2} \qquad (35)$$

5. Performance evaluation

Assume that the background noise power received at each MS is the same, thus we will use σ^2 instead of σ_k^2, hereafter. Without loss of generality, we assume user 1 is the intended (desired) user hereafter. Unless otherwise mentioned, we set $\left(\frac{\eta}{\sigma}\right)^2$ to be 15dB, the number of MS to be 10, and the length of CIR as well as the length of the IR of pre-filter is set to be $L=30$. Note that for a fixed *L*, we generate 500 sets of channel parameters, $\{\mathbf{h}_k\}_{k=1,\ldots,K}$. Each data set is employed for simulation and the result is obtained by taking average of the 500 independent trials. $\{\mathbf{h}_k\}_{k=1,\ldots,K}$ are *i. i. d.* Gaussian random vectors and $\{\|\mathbf{h}_k\|\}_{k=1,\ldots,K}$, which corresponds to the attenuation are generated from $U(0.1,0.9)$.

The "near-far problem" for the multi-user pre-filtering system is verified under two different scenarios: the first case assumes that the desired user is near BS ($\|\mathbf{h}_1\|=0.9$) while the undesired users are far from BS ($\|\mathbf{h}_2\|=\cdots=\|\mathbf{h}_K\|=0.1$); on the other hand, we let $\|\mathbf{h}_1\|=0.1$, $\|\mathbf{h}_2\|=\cdots=\|\mathbf{h}_K\|=0.7$ for the second case. The averaged SINR versus $\left(\frac{\eta}{\sigma}\right)^2$ for both cases of the ZFPF and TRMF schemes are depicted in Fig. 2 (a) and Fig. 2 (b), respectively. It is observed from Fig. 2 (a) that the SINR is severely degraded for case 1 than case 2 especially in high $\left(\frac{\eta}{\sigma}\right)^2$ region, which in term demonstrates the fact that the MS nearest BS is most sensitive to mismatched effect. We also verify from the simulation that mismatched effect makes the SINR for ZFPF scheme insensitive (asymptotically independent) to the variation of $\left(\frac{\eta}{\sigma}\right)^2$. In other words, it is useless to increase transmission power for SINR enhancement under mismatch condition. The result of the TRMF scheme under the same setup is presented in Fig. 2 (b). It is shown that the "near-far problem" has severely deteriorated the performance of the TRMF scheme even in the ideal condition (without mismatch).

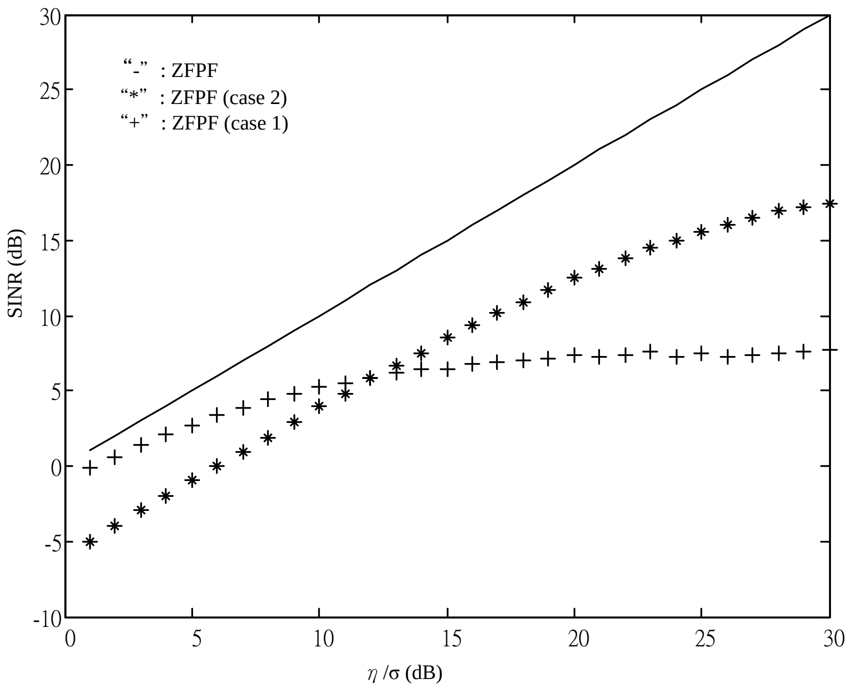
Fig. 3 presents the averaged SINR with respect to the number of MS (denoted by *K*). It is as expected that

SINR degrades as K increases for the TRMF scheme. The ZFPF scheme is essentially robust to MUI since the interference has been completely eliminated, nevertheless, it loses MUI-removal capability under mismatched condition. Specifically, TRMF and ZFPF schemes coincide at K=1 (single user). This is due to the fact that the MF-based scheme is optimum in single user case. In the next simulation, we aim at verifying the effectiveness of the proposed robust processing algorithm on the ZFPF scheme. Note that the values of $\{\delta_k\}_{k=1,\ldots,K}$ in the R-ZFPF scheme are determined according to (36). Fig. 4 compares the SINR performance of the ZFPF and R-ZFPF schemes with respect to $\left(\frac{\eta}{\sigma}\right)^2$ under mismatch condition, where the case without mismatch is also provided for comparison. As shown in the figure, the averaged SINR for ZFPF scheme is severely degraded by mismatch between the downlink and uplink channels. We can also verify from Fig. 4 that the R-ZFPF scheme has significantly enhanced the SINR.

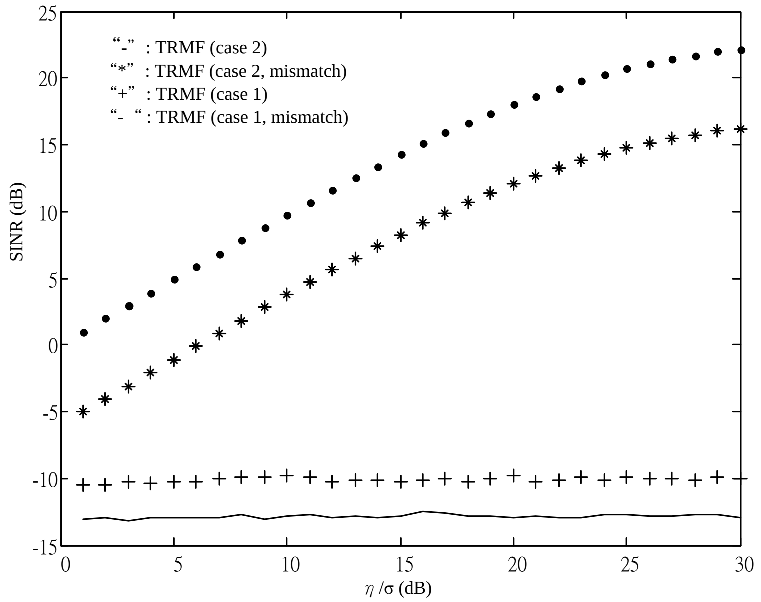
In the final simulation example, we attempt to measure the SINR of TRMF scheme with respect to the pre-filter length, or equivalently, the length of CIR. As shown in Fig. 5, as we increase the pre-filter length from 15 to 110, the average SINR increases as well. It verifies the fact that in the pre-filtering based SDMA communication system, pre-filter length corresponds to the degrees of freedom or processing gain in DS/CDMA system, which determines the capability of mitigating MUI.

6. Conclusions

In this paper, two pre-filtering based SDMA schemes have been proposed and applied in wireless communication system over dense multi-path channel. The benefit of the pre-filtering structure is that it lessens the burden in signal processing of the MS receiver where an extremely simplified receiver is typically required. The TRMF scheme is power efficient, nevertheless, suffers from MUI and near-far problem. On the other hand, the ZFPF scheme uses more power to remove the MUI as well as the near-far effect. The effects of mismatch between the uplink and downlink channels on the ZFPF scheme have also been extensively analyzed. We have shown that the MS that near BS is more sensitive to mismatch. Furthermore, we have developed a simple yet reliable robust ZFPF scheme. We have demonstrated by computer simulation that the proposed robust processing algorithm has comprehensively enhanced the SINR performance of the ZFPF scheme under mismatch condition. The difficulty may arise in applying the pre-filtering technique is in fast fading channel. The MS may require to continuously emitting sounding pulses in order that the transmitter of BS has immediately information of CIRs, which leads to decrease of efficiency.



(a) The ZFPF scheme



(b) The TRMF scheme

Fig. 2: SINR (dB) performance with respect to $\frac{\eta^2}{\sigma^2}$ (dB) for two scenarios: the desired user is near BS while the undesired users are far from BS; the desired user is far from BS while the undesired users are near BS.

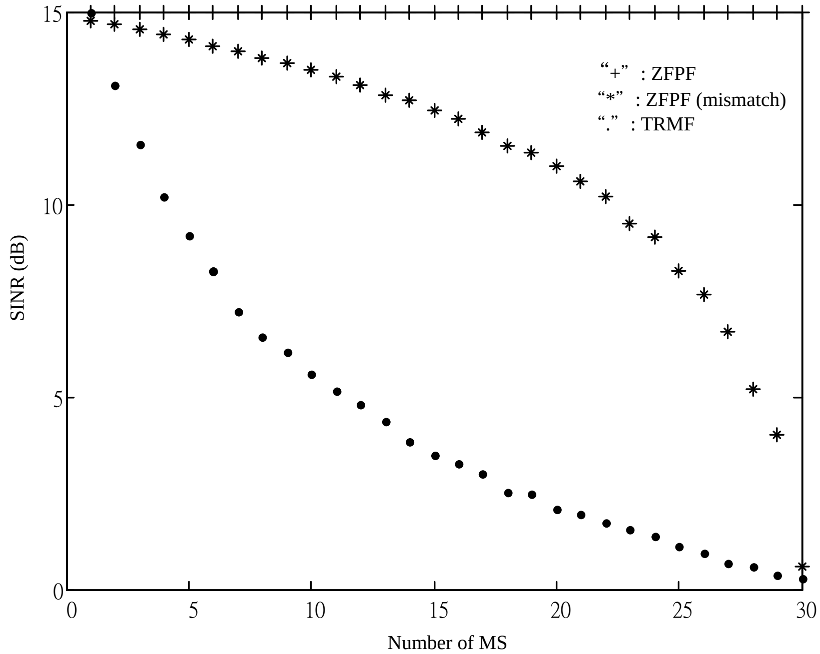


Fig. 3: SINR (dB) performance with respect to the number of MS under mismatch condition

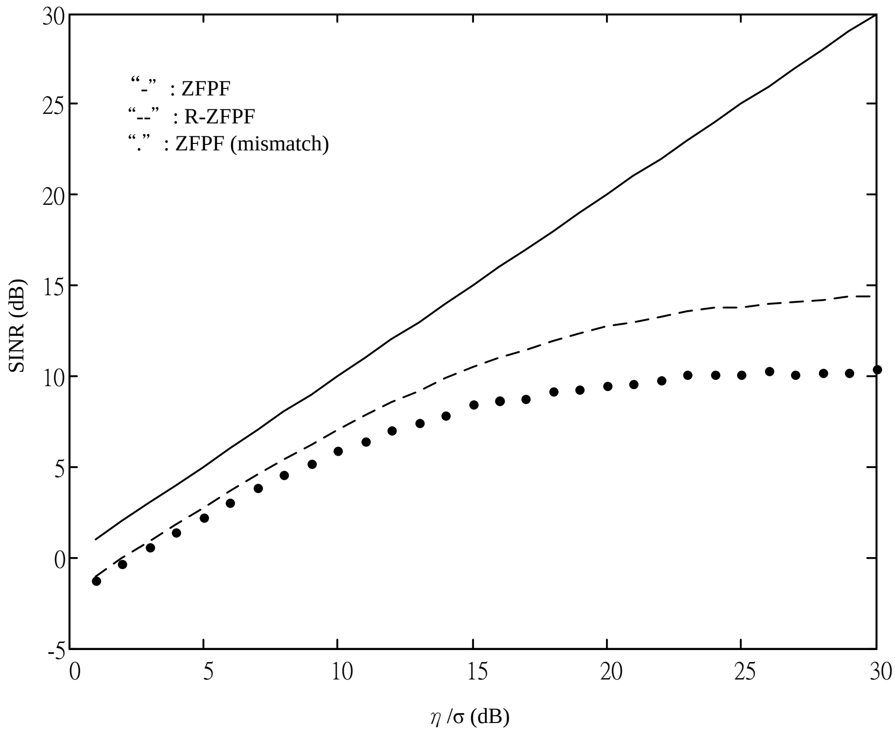


Fig. 4: The SINR (dB) performance of the ZFPF and R-ZFPF schemes with respect to $\left(\frac{\eta}{\sigma}\right)^2$.

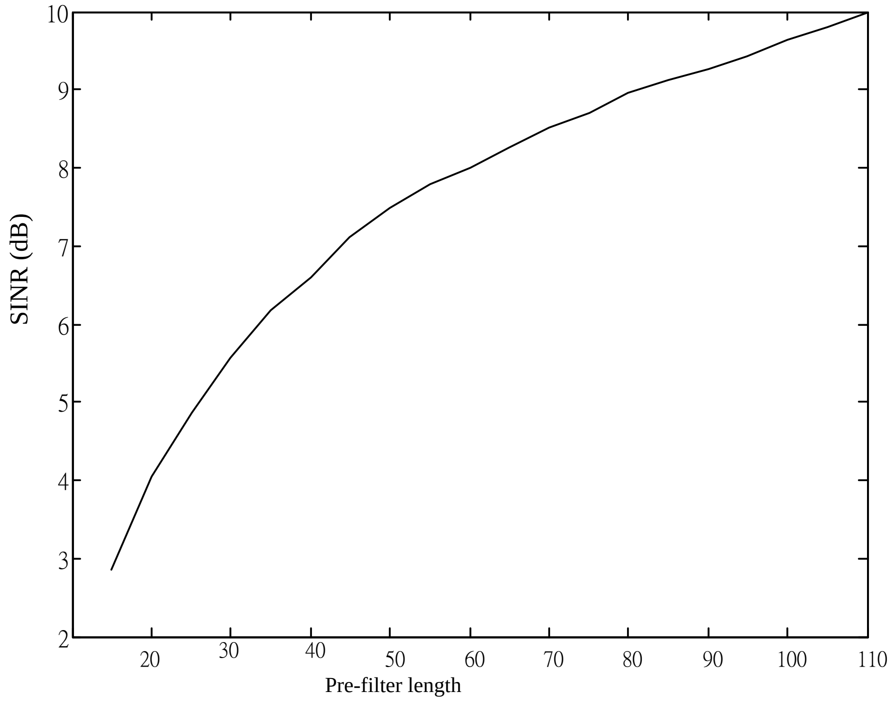


Fig. 5: SINR(dB) performance with respect to pre-filter length of the TRMF scheme.

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時間反轉之空間分割多工技術於多重路徑通道

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摘要

Key words*: *Space Division Multiple Access (SDMA), Time-reversal Matched-filter (TRMF), Zero-forcing Pre-filter (ZFPF), Multiuser interference (MUI).

參考資料[1]提出了時間反轉之空間分割多工技術，在本論文中我們首先將參考資料[1]所提出之架構一般化為預濾波為基礎之空間分割通信系統，其中每個用戶之特徵波形即為基地台至行動台之間的通道響應. 我們提出了兩個預濾波架構：時間反轉匹配濾波器(TRMF)以及零強迫預濾波器(ZFPF). 由於上下鏈之通道響應不盡相同，我們詳盡的分析通道響應不匹配對於 ZFPF 品質所造成的影響，除此之外，為了加強對於不匹配之抵抗能力,我們提出了簡單有效的強健型零強迫預濾波器(R-ZFPF). 我們研究並比較兩種架構之功率損耗以及在不同環境下性能(SINR)之差異. 電腦模擬的結果證明了我們所提出的 R-ZFPF 確能改善不匹配時之性能..

關鍵字*: *空間分割多工技術(SDMA), 時間反轉匹配濾波器(TRMF), 零強迫預濾波器(ZFPF), 多用戶干擾