An Improvement of Delay-Derivative-Dependent Asymptotic Stability Criterion for Interconnected Switched Systems with Time-Varying Delays

Ming-Sheng Yang
Department of Electrical Engineering
Chienkuo Technology University
Changhua, 500, Taiwan, R.O.C.
E-mail:yms@ctu.edu.tw

ABSTRACT

This paper discusses the problem of delay-dependent asymptotic stability for interconnected switched neutral-type systems with time-varying delays. By applying weighting-delay approach, introducing both singular model transformation technique and Finsler's lemma, and constructing an augmented Lyapunov-Krasovskii functional combined with slack matrices, an improved delay-derivative-dependent stability criterion is derived to guarantee the asymptotic stability of above systems. The obtained criterion is formulated in terms of matrix inequalities, which can be efficiently solved via standard numerical software. Two numerical examples are included to show that the proposed method is effective and can provide less conservative results.

Keywords: Interconnected switched neutral-type systems, time-varying delays, weighting-delay approach, singular model transformation, delay-derivative-dependent stability criterion.

1. Introduction

It is well known that a wide class of physical systems in power systems, chemical procedure control systems, navigation systems, automobile speed change system, etc. may be appropriately described by the switched model. Switched systems are a special class of hybrid dynamical systems, which consist of a family of subsystems and a switching law specifying the switching between the subsystems. Recently, there has been increasing interest in the stability problem of switched systems with time delay due to their significance both in theory and applications. To the best of our knowledge, it seems that few people have studied the asymptotic stability problem for continuous-time interconnected switched neutral-type systems with time-varying delays. This has motivated our research.

In this paper, we will give preliminary knowledge for our main result. First of all, consider the following interconnected switched neutral-type system with time-varying delays

$$\dot{x}_i(t)=\sum_{k=1}^{r}\alpha^k(t)\{A_i^k x_i(t)+\overline{A}_i^k x_i(t-d(t))+C_i^k\dot{x}_i(t-d(t))+\sum_{\substack{j=1\\ j\neq i}}^{N}[B_{ij}^k x_j(t)+\overline{B}_{ij}^k x_j(t-d(t))]\} \tag{1a}$$

$$\sum_{k=1}^{r}\alpha^{k}(t)=1,\quad x_i(t)=\varphi_i(t),\qquad t\in[-h,\quad 0] \tag{1b}$$

$$\alpha^{k}(t)=\begin{cases}1, & \text{when the switched system is described by the } k\text{th mode}\\ 0, & \text{otherwise}\end{cases} \tag{1c}$$

where $x_i(t)\in R^{n_i}$ is the state vector of the ith subsystem, A_i^k, $\overline{A}_i^k$, B_{ij}^k, $\overline{B}_{ij}^k$, C_i^k are known constant matrices with appropriate dimensions, $i=1,2,\cdots,N$, $k=1,2,\cdots,r$. The delay $d(t)$ is a time-varying continuous function satisfying $0\le d(t)\le h$ and $\dot{d}(t)\le\mu$. $\varphi_i(t)$ is a given continuous vector-valued initial function.

The following notations will be used throughout this paper. The notation $F>G$ ($F\ge G$) means that the matrix $F-G$ is positive definite (positive semi-definite) for two symmetric matrices F, G. I_i is an identity matrix of appropriate dimensions.

Assumption 1[1]: All the eigenvalues of matrices C_i^k, $i=1,2,\cdots,N$, are inside the unit circle.

Lemma 1[2]: For any real vectors κ_1,κ_2 and any matrix $M>0$ with appropriate dimensions, it follows that

$$2\kappa_1^{\mathrm{T}}\kappa_2\le\kappa_1^{\mathrm{T}}M^{-1}\kappa_1+\kappa_2^{\mathrm{T}}M\kappa_2 \tag{2}$$

Lemma 2[3]: For any symmetric positive definite matrix P and scalars $\lambda>0$, $\delta>1$, the following inequality holds

$$-\int_0^{\lambda}\delta e^{\mathrm{T}}(s)Pe(s)ds\le-\int_0^{\lambda}e^{\mathrm{T}}(s)P\,e(s)ds-\frac{(\delta-1)}{\lambda}\left(\int_0^{\lambda}e(s)\,ds\right)^{\mathrm{T}}P\left(\int_0^{\lambda}e(s)\,ds\right) \tag{3}$$

Lemma 3[4]: For any symmetric positive definite matrix Q and scalars $0\le b_1<b_2$, the following inequality holds

$$-\int_{t-b_2}^{t-b_1}\dot{x}^{\mathrm{T}}(\theta)Q\dot{x}(\theta)d\theta\le-\frac{1}{b_2-b_1}[x(t-b_1)-x(t-b_2)]^{\mathrm{T}}Q\,[x(t-b_1)-x(t-b_2)] \tag{4}$$

Lemma 4(Finsler's lemma)[5]: Consider a vector $\zeta\in R^n$, a symmetric positive definite matrix $S\in R^{n\times n}$ and a matrix $D\in R^{m\times n}$, such that $\mathrm{rank}(D)<n$. The following conditions are equivalent:

(i) $\zeta^{\mathrm{T}}S\zeta<0,\ \forall\zeta$ such that $D\zeta=0$, $\zeta\ne0$ (5a)

(ii) $(D^{\perp})^{\mathrm{T}}SD^{\perp}<0$ (5b)

2. Main Result

In the following theorem, an improved delay-derivative-dependent criterion for asymptotic stability of interconnected switched neutral-type system (1) is proposed in terms of matrix inequalities.

Theorem 1: Under Assumption 1, the interconnected switched neutral-type system (1) is asymptotically stable for $i=1,2,\cdots,N$ and $k=1,2,\cdots,r$, if there exist positive definite matrices J_{11i}, J_{22i}, J_{33i}, J_{44i}, Y_{1i}, Y_{2i}, Z_{11i}, Z_{22i}, Z_{33i}, Z_{44i}, Z_{55i}, Z_{66i}, Z_{77}, P_i, Q_i, R_i, W_{1i}, W_{2i}, W_{3i}, W_{4i}, X_{1i}, X_{2i}, M_{ij}, $\overline{M}_{ij}$, $\hat{M}_{ij}$, $\tilde{M}_{ij}$, real matrices H_i, J_{12i}, J_{13i}, J_{14i}, J_{23i}, J_{24i}, J_{34i}, Z_{12i}, Z_{13i}, Z_{14i}, Z_{15i}, Z_{16i}, Z_{17i}, Z_{23i}, Z_{24i}, Z_{25i}, Z_{26i}, Z_{27i}, Z_{34i}, Z_{35i}, Z_{36i}, Z_{37i}, Z_{45i}, Z_{46i}, Z_{47i}, Z_{56i}, Z_{57i}, Z_{67i}, and scalars $0<\rho<1$, $\delta_i>1$ such that the following conditions hold

$$\begin{bmatrix}Y_{1i} & H_i\\ H_i^{\mathrm{T}} & Y_{2i}\end{bmatrix}>0 \tag{6a}$$

$$\begin{bmatrix} J_{11i} & J_{12i} & J_{13i} & J_{14i} \\ J_{12i}^{\mathrm{T}} & J_{22i} & J_{23i} & J_{24i} \\ J_{13i}^{\mathrm{T}} & J_{23i}^{\mathrm{T}} & J_{33i} & J_{34i} \\ J_{14i}^{\mathrm{T}} & J_{24i}^{\mathrm{T}} & J_{34i}^{\mathrm{T}} & J_{44i} \end{bmatrix} > 0 \tag{6b}$$

$$Z_i = \begin{bmatrix} Z_{11i} & Z_{12i} & Z_{13i} & Z_{14i} & Z_{15i} & Z_{16i} & Z_{17i} \\ Z_{12i}^{\mathrm{T}} & Z_{22i} & Z_{23i} & Z_{24i} & Z_{25i} & Z_{26i} & Z_{27i} \\ Z_{13i}^{\mathrm{T}} & Z_{23i}^{\mathrm{T}} & Z_{33i} & Z_{34i} & Z_{35i} & Z_{36i} & Z_{37i} \\ Z_{14i}^{\mathrm{T}} & Z_{24i}^{\mathrm{T}} & Z_{34i}^{\mathrm{T}} & Z_{44i} & Z_{45i} & Z_{46i} & Z_{47i} \\ Z_{15i}^{\mathrm{T}} & Z_{25i}^{\mathrm{T}} & Z_{35i}^{\mathrm{T}} & Z_{45i}^{\mathrm{T}} & Z_{55i} & Z_{56i} & Z_{57i} \\ Z_{16i}^{\mathrm{T}} & Z_{26i}^{\mathrm{T}} & Z_{36i}^{\mathrm{T}} & Z_{46i}^{\mathrm{T}} & Z_{56i}^{\mathrm{T}} & Z_{66i} & Z_{67i} \\ Z_{17i}^{\mathrm{T}} & Z_{27i}^{\mathrm{T}} & Z_{37i}^{\mathrm{T}} & Z_{47i}^{\mathrm{T}} & Z_{57i}^{\mathrm{T}} & Z_{67i}^{\mathrm{T}} & Z_{77i} \end{bmatrix} > 0 \tag{6c}$$

$$X_{2i} + (1-\mu)X_{1i} - Z_{77i} > 0 \tag{6d}$$

$$(D_i^{\perp})^{\mathrm{T}} \Pi_i D_i^{\perp} < 0 \tag{6e}$$

where

$$D_i^{\perp} = \begin{bmatrix} I_i & I_i & 0 & 0 & I_i \\ 0 & I_i & 0 & 0 & 0 \\ I_i & 0 & 0 & 0 & 0 \\ I_i & I_i & 0 & 0 & 0 \\ 0 & 0 & I_i & 0 & 0 \\ 0 & 0 & 0 & I_i & 0 \\ 0 & 0 & 0 & 0 & I_i \end{bmatrix} \tag{7a}$$

$$\Pi_i = \begin{bmatrix} \Pi_{11i} & \Pi_{12i} & \Pi_{13i} & \Pi_{14i} & \Pi_{15i} & \Pi_{16i} & \Pi_{17i} \\ \Pi_{12i}^{\mathrm{T}} & \Pi_{22i} & \Pi_{23i} & \Pi_{24i} & \Pi_{25i} & \Pi_{26i} & \Pi_{27i} \\ \Pi_{13i}^{\mathrm{T}} & \Pi_{23i}^{\mathrm{T}} & \Pi_{33i} & \Pi_{34i} & \Pi_{35i} & \Pi_{36i} & \Pi_{37i} \\ \Pi_{14i}^{\mathrm{T}} & \Pi_{24i}^{\mathrm{T}} & \Pi_{34i}^{\mathrm{T}} & \Pi_{44i} & \Pi_{45i} & \Pi_{46i} & \Pi_{47i} \\ \Pi_{15i}^{\mathrm{T}} & \Pi_{25i}^{\mathrm{T}} & \Pi_{35i}^{\mathrm{T}} & \Pi_{45i}^{\mathrm{T}} & \Pi_{55i} & \Pi_{56i} & \Pi_{57i} \\ \Pi_{16i}^{\mathrm{T}} & \Pi_{26i}^{\mathrm{T}} & \Pi_{36i}^{\mathrm{T}} & \Pi_{46i}^{\mathrm{T}} & \Pi_{56i}^{\mathrm{T}} & \Pi_{66i} & \Pi_{67i} \\ \Pi_{17i}^{\mathrm{T}} & \Pi_{27i}^{\mathrm{T}} & \Pi_{37i}^{\mathrm{T}} & \Pi_{47i}^{\mathrm{T}} & \Pi_{57i}^{\mathrm{T}} & \Pi_{67i}^{\mathrm{T}} & \Pi_{77i} \end{bmatrix} \tag{7b}$$

$$\Pi_{11i} = R_i A_i^k + (A_i^k)^{\mathrm{T}} R_i - \frac{1}{\rho h}[X_{2i} + (1-\mu)X_{1i} - Z_{66i}] + W_{1i} + W_{2i} + J_{11i} + W_{4i} + Y_{1i} + \rho h Z_{11i}$$
$$+ \sum_{\substack{j=1 \\ j\neq i}}^{N} \{ R_i [B_{ij}^k M_{ij} (B_{ij}^k)^{\mathrm{T}} + \overline{B}_{ij}^k \overline{M}_{ij} (\overline{B}_{ij}^k)^{\mathrm{T}}] R_i + M_{ji}^{-1} + \hat{M}_{ji}^{-1} \} \tag{7c}$$

$$\Pi_{12i} = P_i - R_i + (A_i^k)^{\mathrm{T}} Q_i + J_{12i} + \rho h Z_{12i}, \Pi_{13i} = R_i \overline{A}_i^k + J_{13i} + \rho h Z_{13i} \tag{7d}$$

$$\Pi_{14i} = \rho h Z_{14i} + \frac{1}{\rho h}[X_{2i} + (1-\mu)X_{1i} - Z_{77i}] \tag{7e}$$

$$\Pi_{15i} = R_i C_i^k + \rho h Z_{15i}, \Pi_{16i} = \rho h Z_{16i}, \Pi_{17i} = J_{14i} + Z_{17i} \tag{7f}$$

$$\Pi_{22i} = W_{3i} + Y_{2i} + J_{22i} + \rho h Z_{22i} - 2Q_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} Q_i [B_{ij}^k \hat{M}_{ij} (B_{ij}^k)^{\mathrm{T}} + \overline{B}_{ij}^k \tilde{M}_{ij} (\overline{B}_{ij}^k)^{\mathrm{T}}] Q_i \tag{7g}$$

$$\Pi_{23i} = Q_i \overline{A}_i^k + J_{23i} + \rho h Z_{23i}, \ \Pi_{24i} = \rho h Z_{24i}, \Pi_{25i} = Q_i C_i^k + \rho h Z_{25i}, \Pi_{26i} = \rho h Z_{26i}, \Pi_{27i} = J_{24i} + Z_{27i} \tag{7h}$$

$$\Pi_{33i} = J_{33i} + \rho h Z_{33i} - (1-\mu)(W_{1i} + Y_{1i}) + \sum_{\substack{j=1 \\ j\neq i}}^{N} (\overline{M}_{ji}^{-1} + \tilde{M}_{ji}^{-1}) \tag{7i}$$

$$\Pi_{34i} = \rho h Z_{34i}, \Pi_{35i} = -(1-\mu)H_i + \rho h Z_{35i}, \Pi_{36i} = \rho h Z_{36i}, \ \Pi_{37i} = J_{34i} + Z_{37i} \quad (7j)$$

$$\Pi_{44i} = -\frac{1}{\rho h}[X_{2i} + (1-\mu)X_{1i} - Z_{77i}] - \frac{\delta_i}{h}X_{2i} - (1-\rho\mu)W_{2i} + \rho h Z_{44i} \quad (7k)$$

$$\Pi_{45i} = \rho h Z_{45i}, \ \Pi_{46i} = \rho h Z_{46i} + \frac{\delta_i}{h}X_{2i}, \ \Pi_{47i} = Z_{47i} \quad (7l)$$

$$\Pi_{55i} = \rho h Z_{55i} - (1-\mu)(W_{3i} + Y_{2i}), \Pi_{56i} = \rho h Z_{56i}, \ \Pi_{57i} = Z_{57i} \quad (7m)$$

$$\Pi_{66i} = \rho h Z_{66i} - \frac{\delta_i}{h}X_{2i}, \Pi_{67i} = Z_{67i}, \Pi_{77i} = J_{44i} - \frac{(\delta_i - 1)}{\rho h}X_{2i} \quad (7n)$$

Proof: Based on singular model transformation [6], system (1) can be written as

$$\dot{x}_i(t) = y_i(t) \quad (8a)$$

$$0 = \sum_{k=1}^{r} \alpha^k(t)\{-y_i(t) + C_i^k y_i(t-d(t)) + A_i^k x_i(t) + \overline{A}_i^k x_i(t-d(t)) + \sum_{\substack{j=1 \\ j\neq i}}^{N} [B_{ij}^k x_j(t) + \overline{B}_{ij}^k x_j(t-d(t))]\} \quad (8b)$$

By means of the idea of [7] and [8], we use the following Lyapunov-Krasovskii functional to derive the stability criterion

$$V(t) = \sum_{i=1}^{N}[V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t) + V_{5i}(t) + V_{6i}(t) + V_{7i}(t) + V_{8i}(t) + V_{9i}(t) + V_{10i}(t)] \quad (9)$$

where

$$V_{1i}(t) = [x_i^{\mathrm{T}}(t) \quad y_i^{\mathrm{T}}(t)]\begin{bmatrix} I_i & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} P_i & 0 \\ R_i & Q_i \end{bmatrix}\begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \quad (10a)$$

$$V_{2i}(t) = \int_{t-d(t)}^{t} x_i^{\mathrm{T}}(s) W_{1i} x_i(s) ds \quad (10b)$$

$$V_{3i}(t) = \int_{t-\rho d(t)}^{t} x_i^{\mathrm{T}}(s) W_{2i} x_i(s) ds \quad (10c)$$

$$V_{4i}(t) = \int_{t-d(t)}^{t} y_i^{\mathrm{T}}(s) W_{3i} y_i(s) ds \quad (10d)$$

$$V_{5i}(t) = \int_{t-h}^{t} x_i^{\mathrm{T}}(s) W_{4i} x_i(s) ds \quad (10e)$$

$$V_{6i}(t) = \int_{t-d(t)}^{t} \begin{bmatrix} x_i(s) \\ y_i(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Y_{1i} & H_i \\ H_i^{\mathrm{T}} & Y_{2i} \end{bmatrix}\begin{bmatrix} x_i(s) \\ y_i(s) \end{bmatrix} ds \quad (10f)$$

$$V_{7i}(t) = \int_{-d(t)}^{0} \int_{t+\theta}^{t} y_i^{\mathrm{T}}(s) X_{1i} y_i(s) ds d\theta \quad (10g)$$

$$V_{8i}(t) = \int_{-h}^{0} \int_{t+\theta}^{t} \delta_i y_i^{\mathrm{T}}(s) X_{2i} y_i(s) ds d\theta \quad (10h)$$

$$V_{9i}(t) = \int_{0}^{t} \int_{\theta-\rho d(\theta)}^{\theta} e_i^{\mathrm{T}}(\theta, s) Z_i e_i(\theta, s) ds d\theta \quad (10i)$$

$$V_{10i}(t)=\int_0^t\begin{bmatrix}x_i(\theta)\\ y_i(\theta)\\ x_i(\theta-d(\theta))\\ \int_{\theta-\rho d(\theta)}^{\theta}y_i(s)ds\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}J_{11i} & J_{12i} & J_{13i} & J_{14i}\\ J_{12i}^{\mathrm{T}} & J_{22i} & J_{23i} & J_{24i}\\ J_{13i}^{\mathrm{T}} & J_{23i}^{\mathrm{T}} & J_{33i} & J_{34i}\\ J_{14i}^{\mathrm{T}} & J_{24i}^{\mathrm{T}} & J_{34i}^{\mathrm{T}} & J_{44i}\end{bmatrix}\begin{bmatrix}x_i(\theta)\\ y_i(\theta)\\ x_i(\theta-d(\theta))\\ \int_{\theta-\rho d(\theta)}^{\theta}y_i(s)ds\end{bmatrix}d\theta \tag{10j}$$

where $e_i(\theta,s)=[x_i^{\mathrm{T}}(\theta)\ y_i^{\mathrm{T}}(\theta)\ x_i^{\mathrm{T}}(\theta-d(\theta))\ x_i^{\mathrm{T}}(\theta-\rho d(\theta))\ y_i^{\mathrm{T}}(\theta-d(\theta))\ x_i^{\mathrm{T}}(\theta-h)\ y_i^{\mathrm{T}}(s)]^{\mathrm{T}}$ and matrix Z_i is defined in (6c).

Taking the time derivative of $V(t)$ along the trajectories of system (1) and noting that $0\le d(t)\le h$ and $\dot{d}(t)\le\mu$, it yields

$$\dot{V}(t)=\sum_{i=1}^{N}[\dot{V}_{1i}(t)+\dot{V}_{2i}(t)+\dot{V}_{3i}(t)+\dot{V}_{4i}(t)+\dot{V}_{5i}(t)+\dot{V}_{6i}(t)+\dot{V}_{7i}(t)+\dot{V}_{8i}(t)+\dot{V}_{9i}(t)+\dot{V}_{10i}(t)] \tag{11}$$

where

$$\dot{V}_{1i}(t)=2[x_i^{\mathrm{T}}(t)\quad y_i^{\mathrm{T}}(t)]\begin{bmatrix}P_i & R_i\\ 0 & Q_i\end{bmatrix}$$

$$\times\begin{bmatrix}y_i(t)\\ \left(\begin{array}{l}\sum_{k=1}^{r}\alpha^k(t)\{-y_i(t)+C_i^k y_i(t-d(t))\\ \quad+A_i^k x_i(t)+\overline{A}_i^k x_i(t-d(t))\\ \quad+\sum_{j=1,\,j\neq i}^{N}[B_{ij}^k x_j(t)+\overline{B}_{ij}^k x_j(t-d(t))]\}\end{array}\right)\end{bmatrix} \tag{12a}$$

$$\dot{V}_{2i}(t)\le x_i^{\mathrm{T}}(t)W_{1i}x_i(t)-(1-\mu)x_i^{\mathrm{T}}(t-d(t))W_{1i}x_i(t-d(t)) \tag{12b}$$

$$\dot{V}_{3i}(t)\le x_i^{\mathrm{T}}(t)W_{2i}x_i(t)-(1-\rho\mu)x_i^{\mathrm{T}}(t-\rho d(t))W_{2i}x_i(t-\rho d(t)) \tag{12c}$$

$$\dot{V}_{4i}(t)\le y_i^{\mathrm{T}}(t)W_{3i}y_i(t)-(1-\mu)y_i^{\mathrm{T}}(t-d(t))W_{3i}y_i(t-d(t)) \tag{12d}$$

$$\dot{V}_{5i}(t)\le x_i^{\mathrm{T}}(t)W_{4i}x_i(t)-x_i^{\mathrm{T}}(t-h)W_{4i}x_i(t-h) \tag{12e}$$

$$\dot{V}_{6i}(t)\le\begin{bmatrix}x_i(t)\\ y_i(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Y_{1i} & H_i\\ H_i^{\mathrm{T}} & Y_{2i}\end{bmatrix}\begin{bmatrix}x_i(t)\\ y_i(t)\end{bmatrix}-(1-\mu)\begin{bmatrix}x_i(t-d(t))\\ y_i(t-d(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Y_{1i} & H_i\\ H_i^{\mathrm{T}} & Y_{2i}\end{bmatrix}\begin{bmatrix}x_i(t-d(t))\\ y_i(t-d(t))\end{bmatrix} \tag{12f}$$

$$\dot{V}_{7i}(t)\le hy_i^{\mathrm{T}}(t)X_{1i}y_i(t)-(1-\mu)\int_{t-d(t)}^{t}y_i^{\mathrm{T}}(s)X_{1i}y_i(s)ds \tag{12g}$$

$$\dot{V}_{8i}(t)\le h\delta_i y_i^{\mathrm{T}}(t)X_{2i}y_i(t)-\int_{t-\rho d(t)}^{t}\delta_i y_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds-\int_{t-h}^{t-\rho d(t)}\delta_i y_i^{\mathrm{T}}(s)X_{2i}y_i(s)ds \tag{12h}$$

$$\dot{V}_{9i}(t)=\rho d(t)\begin{bmatrix}x_i(t)\\ y_i(t)\\ x_i(t-d(t))\\ x_i(t-\rho d(t))\\ y_i(t-d(t))\\ x_i(t-h)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_{11i} & Z_{12i} & Z_{13i} & Z_{14i} & Z_{15i} & Z_{16i}\\ Z_{12i}^{\mathrm{T}} & Z_{22i} & Z_{23i} & Z_{24i} & Z_{25i} & Z_{26i}\\ Z_{13i}^{\mathrm{T}} & Z_{23i}^{\mathrm{T}} & Z_{33i} & Z_{34i} & Z_{35i} & Z_{36i}\\ Z_{14i}^{\mathrm{T}} & Z_{24i}^{\mathrm{T}} & Z_{34i}^{\mathrm{T}} & Z_{44i} & Z_{45i} & Z_{46i}\\ Z_{15i}^{\mathrm{T}} & Z_{25i}^{\mathrm{T}} & Z_{35i}^{\mathrm{T}} & Z_{45i}^{\mathrm{T}} & Z_{55i} & Z_{56i}\\ Z_{16i}^{\mathrm{T}} & Z_{26i}^{\mathrm{T}} & Z_{36i}^{\mathrm{T}} & Z_{46i}^{\mathrm{T}} & Z_{56i}^{\mathrm{T}} & Z_{66i}\end{bmatrix}\begin{bmatrix}x_i(t)\\ y_i(t)\\ x_i(t-d(t))\\ x_i(t-\rho d(t))\\ y_i(t-d(t))\\ x_i(t-h)\end{bmatrix}$$

$$+\int_{t-\rho d(t)}^{t}2[x_i^{\mathrm{T}}(t)Z_{17i}+y_i^{\mathrm{T}}(t)Z_{27i}]y_i(s)ds+\int_{t-\rho d(t)}^{t}2[x_i^{\mathrm{T}}(t-d(t))Z_{37i}+x_i^{\mathrm{T}}(t-\rho d(t))Z_{47i}]y_i(s)ds$$

$$+\int_{t-\rho d(t)}^{t}2[y_i^{\mathrm{T}}(t-d(t))Z_{57i}+x_i^{\mathrm{T}}(t-h)Z_{67i}]y_i(s)ds+\int_{t-\rho d(t)}^{t}y_i^{\mathrm{T}}(s)Z_{77i}y_i(s)ds \tag{12i}$$

$$\dot{V}_{10\,i}(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \\ x_i(t-d(t)) \\ \int_{t-\rho d(t)}^{t} y_i(s)ds \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} J_{11i} & J_{12i} & J_{13i} & J_{14i} \\ J_{12i}^{\mathrm{T}} & J_{22i} & J_{23i} & J_{24i} \\ J_{13i}^{\mathrm{T}} & J_{23i}^{\mathrm{T}} & J_{33i} & J_{34i} \\ J_{14i}^{\mathrm{T}} & J_{24i}^{\mathrm{T}} & J_{34i}^{\mathrm{T}} & J_{44i} \end{bmatrix} \begin{bmatrix} x_i(t) \\ y_i(t) \\ x_i(t-d(t)) \\ \int_{t-\rho d(t)}^{t} y_i(s)ds \end{bmatrix} \tag{12j}$$

Applying Lemma 1, we have

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} 2x_i^{\mathrm{T}}(t)R_i B_{ij}^k x_j(t) \\ &\leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} [x_i^{\mathrm{T}}(t)R_i B_{ij}^k M_{ij}(B_{ij}^k)^{\mathrm{T}} R_i x_i(t) + x_j^{\mathrm{T}}(t) M_{ij}^{-1} x_j(t)] \\ &= \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} x_i^{\mathrm{T}}(t)[R_i B_{ij}^k M_{ij}(B_{ij}^k)^{\mathrm{T}} R_i + M_{ji}^{-1}]\, x_i(t) \end{aligned} \tag{13a}$$

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} 2y_i^{\mathrm{T}}(t)Q_i B_{ij}^k x_j(t) \\ &\leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} [y_i^{\mathrm{T}}(t)Q_i B_{ij}^k \hat{M}_{ij}(B_{ij}^k)^{\mathrm{T}} Q_i y_i(t) + x_j^{\mathrm{T}}(t)\hat{M}_{ij}^{-1} x_j(t)] \\ &= \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} [y_i^{\mathrm{T}}(t)Q_i B_{ij}^k \hat{M}_{ij}(B_{ij}^k)^{\mathrm{T}} Q_i y_i(t) + x_i^{\mathrm{T}}(t)\hat{M}_{ji}^{-1} x_i(t)] \end{aligned} \tag{13b}$$

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} 2x_i^{\mathrm{T}}(t)R_i \overline{B}_{ij}^k x_j(t-d(t)) \\ &\leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} [x_i^{\mathrm{T}}(t)R_i \overline{B}_{ij}^k \overline{M}_{ij}(\overline{B}_{ij}^k)^{\mathrm{T}} R_i x_i(t) + x_j^{\mathrm{T}}(t-d(t))\overline{M}_{ij}^{-1} x_j(t-d(t))] \\ &= \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} [x_i^{\mathrm{T}}(t)R_i \overline{B}_{ij}^k \overline{M}_{ij}(\overline{B}_{ij}^k)^{\mathrm{T}} R_i x_i(t) + x_i^{\mathrm{T}}(t-d(t))\overline{M}_{ji}^{-1} x_i(t-d(t))] \end{aligned} \tag{13c}$$

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} 2y_i^{\mathrm{T}}(t)Q_i \overline{B}_{ij}^k x_j(t-d(t))$$

$$\leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} [y_i^{\mathrm{T}}(t)Q_i \overline{B}_{ij}^k \tilde{M}_{ij}(\overline{B}_{ij}^k)^{\mathrm{T}} Q_i y_i(t) + x_j^{\mathrm{T}}(t-d(t))\overline{M}_{ij}^{-1} x_j(t-d(t))]$$

$$= \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} [y_i^{\mathrm{T}}(t) Q_i \overline{B}_{ij}^k \tilde{M}_{ij} (\overline{B}_{ij}^k)^{\mathrm{T}} Q_i y_i(t) + x_i^{\mathrm{T}}(t-d(t)) \tilde{M}_{ji}^{-1} x_i(t-d(t))] \tag{13d}$$

According to Lemma 2 and using the idea of [9], we get

$$-\int_{t-\rho d(t)}^{t} \delta_i y_i^{\mathrm{T}}(s) X_{2i} y_i(s) ds$$

$$\leq -\frac{(\delta_i - 1)}{\rho h} \left[\int_{t-\rho d(t)}^{t} y_i(s) ds \right]^{\mathrm{T}} X_{2i} \left[\int_{t-\rho d(t)}^{t} y_i(s) ds \right] - \int_{t-\rho d(t)}^{t} y_i^{\mathrm{T}}(s) X_{2i} y_i(s) ds \tag{14}$$

From (6d), (12g), (12i), (14) and Lemma 3, we have

$$-\int_{t-\rho d(t)}^{t} y_i^{\mathrm{T}}(s) [X_{2i} + (1-\mu) X_{1i} - Z_{77i}] y_i(s) ds$$

$$\leq -\frac{1}{\rho h} [x_i(t) - x_i(t-\rho d(t))]^{\mathrm{T}} [X_{2i} + (1-\mu) X_{1i} - Z_{77i}] [x_i(t) - x_i(t-\rho d(t))] \tag{15a}$$

$$-\int_{t-h}^{t-\rho d(t)} \delta_i y_i^{\mathrm{T}}(s) X_{2i} y_i(s) ds$$

$$\leq -\frac{\delta_i}{h} [x_i(t-\rho d(t)) - x_i(t-h)]^{\mathrm{T}} X_{2i} [x_i(t-\rho d(t)) - x_i(t-h)] \tag{15b}$$

From (11) – (15), we obtain

$$\dot{V}(t) \leq \sum_{i=1}^{N} \sum_{k=1}^{r} \alpha^k(t) \omega_i^{\mathrm{T}}(t) \Pi_i \omega_i(t) \tag{16}$$

where $\omega_i(t) = [x_i^{\mathrm{T}}(t) \quad y_i^{\mathrm{T}}(t) \quad x_i^{\mathrm{T}}(t-d(t)) \quad x_i^{\mathrm{T}}(t-\rho d(t)) \quad y_i^{\mathrm{T}}(t-d(t)) \quad x_i^{\mathrm{T}}(t-h) \quad (\int_{t-\rho d(t)}^{t} y_i(s) ds)^{\mathrm{T}}]^{\mathrm{T}}$ and matrix Π_i is defined in (7b).

Based on Leibniz-Newton formula, we get

$$x_i(t) - x_i(t-\rho d(t)) - \int_{t-\rho d(t)}^{t} y_i(s) ds = 0 \tag{17}$$

This means

$$D_i \omega_i(t) = 0 \tag{18}$$

where $D_i = [I_i \quad 0 \quad 0 \quad -I_i \quad 0 \quad 0 \quad -I_i]$.

From Lemma 4, it is seen that $\omega_i^{\mathrm{T}}(t) \Pi_i \omega_i(t) < 0$ is equivalent to inequality (6e). Obviously, if inequality (6e) holds, then $\dot{V}(t) < 0$, which ensures that system (8) is asymptotically stable [1]. It means that system (1) is asymptotically stable, too. The proof is completed.

3. Numerical Examples

In this section, two examples are given to show the benefits of our result.

Example 1: Consider the following interconnected switched time-varying-delay system composed of two individual switched systems:

Switched system 1 (k = 1):

$$\dot{x}_1(t)=\begin{bmatrix}-5.5 & 0\\ 0 & -3.3\end{bmatrix}x_1(t)+\begin{bmatrix}-0.5 & 0.4\\ 0.1 & -0.3\end{bmatrix}x_1(t-d(t))+\begin{bmatrix}0.2 & 1\\ 0.5 & 0.2\end{bmatrix}x_2(t)+\begin{bmatrix}0.5 & 0\\ 0.1 & 0\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0 & -0.1\\ -0.2 & 0\end{bmatrix}x_3(t)+\begin{bmatrix}-1 & 0\\ 0.1 & 0\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_2(t)=\begin{bmatrix}-8.3 & 0\\ 0 & -6.3\end{bmatrix}x_2(t)+\begin{bmatrix}0.7 & 0\\ -0.5 & -1\end{bmatrix}x_2(t-d(t))+\begin{bmatrix}0.2 & 0.3\\ 0.1 & 0.7\end{bmatrix}x_1(t)+\begin{bmatrix}1.1 & 0.2\\ 0.3 & 0\end{bmatrix}x_1(t-d(t))$$
$$+\begin{bmatrix}1.1 & 0.1\\ 0.3 & 0.2\end{bmatrix}x_3(t)+\begin{bmatrix}-1 & 0.5\\ 0.7 & 0.1\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_3(t)=\begin{bmatrix}-9.2 & 0\\ 0 & -7.2\end{bmatrix}x_3(t)+\begin{bmatrix}-1 & 1\\ 0.5 & -3\end{bmatrix}x_3(t-d(t))+\begin{bmatrix}0 & 0.4\\ 1 & 0\end{bmatrix}x_2(t)+\begin{bmatrix}0.1 & 0\\ 0.2 & 1\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0.1 & 0.5\\ 0 & 0.1\end{bmatrix}x_1(t)+\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}x_1(t-d(t)) \tag{19a}$$

Switched system 2 (k = 2):

$$\dot{x}_1(t)=\begin{bmatrix}-2.5 & 0\\ 0 & -3.5\end{bmatrix}x_1(t)+\begin{bmatrix}-0.1 & 0\\ 0.5 & -0.1\end{bmatrix}x_1(t-d(t))+\begin{bmatrix}0.1 & 0.2\\ 0.1 & 0\end{bmatrix}x_2(t)+\begin{bmatrix}0 & 0\\ 0.2 & 0.1\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}x_3(t)+\begin{bmatrix}0 & 0.1\\ 0.2 & 0\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_2(t)=\begin{bmatrix}-3.6 & 0\\ 0 & -5\end{bmatrix}x_2(t)+\begin{bmatrix}-0.5 & 0\\ 0 & -0.5\end{bmatrix}x_2(t-d(t))+\begin{bmatrix}0 & 0.1\\ 0.5 & 0\end{bmatrix}x_1(t)+\begin{bmatrix}0 & 0.5\\ 0.1 & 0\end{bmatrix}x_1(t-d(t))$$
$$+\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}x_3(t)+\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix}x_3(t-d(t))$$

$$\dot{x}_3(t)=\begin{bmatrix}-7 & 0\\ 0 & -2.6\end{bmatrix}x_3(t)+\begin{bmatrix}-0.5 & 0\\ 0 & -0.5\end{bmatrix}x_3(t-d(t))+\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}x_2(t)+\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix}x_2(t-d(t))$$
$$+\begin{bmatrix}0 & 0\\ 0.2 & 0\end{bmatrix}x_1(t)+\begin{bmatrix}0 & 0.3\\ 0 & 0.2\end{bmatrix}x_1(t-d(t)) \tag{19b}$$

Our purpose in example 1 is to find the maximum allowed delay h of $d(t)$ satisfying $\dot{d}(t)\le\mu,$ such that the switching system (19) is asymptotically stable. A comparison between our Theorem 1 and the method of [10] is shown in Table 1, which also displays the maximum allowed delay h and its time derivative μ for guaranteeing the asymptotic stability of system (19). Obviously, it can be seen that the weighting-delay-dependent stability criterion in this paper is less conservative than one given by [10].

Table 1. Allowable delay bound *h* for different μ

μ	*h* ([10])	*h* (Our Theorem 1)
0.5	Fail	6.5631
1.0	Fail	5.7382
1.5	Fail	4.6297
2.0	Fail	3.9156
2.5	Fail	2.8353

Example 2: Consider the following switched systems with time-varying delay

Switched system 1:

$$\dot{x}(t)=\begin{bmatrix}-5.5 & 0\\ 0 & -1.5\end{bmatrix}x(t)+\begin{bmatrix}-0.1 & 0.2\\ 0.1 & 0.5\end{bmatrix}x(t-d(t)) \tag{20a}$$

Switched system 2:

$$\dot{x}(t)=\begin{bmatrix}-2.2 & 0\\ 0 & -7.7\end{bmatrix}x(t)+\begin{bmatrix}0.1 & 0.5\\ 0.5 & 0.1\end{bmatrix}x(t-d(t)) \tag{20b}$$

Our purpose in example 2 is to find the maximum allowed delay h of $d(t)$ satisfying $\dot{d}(t)\le\mu,$ such that the switching system (20) is asymptotically stable. A comparison between our Theorem 1 and the methods of [11], [12] and [13] is shown in Table 2, which also displays the maximum allowed delay *h* and its time derivative μ for guaranteeing the asymptotic stability of system (20). It is clear that our new method produces better results than those in [11], [12] and [13].

Table 2. Allowable delay bound *h* for different μ

μ	*h* ([11])	*h* ([12])	*h* ([13])	*h* (Our Theorem 1)
0.1	1.3519	2.5381	3.3215	9.3129
0.3	0.6287	1.9236	2.6738	8.9153
0.7	0.4093	1.0153	1.3596	7.5816
0.9	0.3182	0.6928	0.9361	6.6187
1.1	0.1016	0.3527	0.5329	5.8652

4. Conclusion

A class of interconnected switched neutral-type system with time-varying delays has been investigated in this paper. By means of an augmented Lyapunov-Krasovskii functional form combined with slack matrices, singular model transformation technique, Finsler's lemma and weighting-delay approach, an improved delay-derivative-dependent stability criterion is derived in terms of matrix inequalities. Two numerical examples are given to show the effectiveness and benefits of the proposed criterion.

References

[1] Kolmanovskii, V., & Myshkis, A. (1992). *Applied Theory of Functional Differential Equations.* Boston, MA: Kluwer.

[2] Cao, Y. Y., Sun, Y. X., & Cheng, C. (1998). Delay-dependent robust stabilization of uncertain systems with multiple state delays. *IEEE Trans. Automat. Control, 43*(11), 1608-1612.

[3] Kwon, O. M., & Park, J. H. (2004). On improved delay-dependent robust control for uncertain time-delay systems. *IEEE Trans. Automat. Control, 49*(11), 1991-1995.

[4] Gu, K., Kharitonov, V. L., & Chen, J. (2003). *Stability of Time-Delay Systems*. Boston, MA: Birkhauser.

[5] De Oliveira, M. C., & Skelton, R. E. (2001). Stability tests for constrained linear systems. in *Perspectives in Robust Control*, S. O. Reza Moheimani (Ed.), *series Lecture Notes in Control and Information Sciences* (pp. 241-257). New York: Springer-Verlag.

[6] Fridman, E., & Shaked, U. (2002). A descriptor system approach to H_∞ control of linear time-delay systems. *IEEE Trans. Automat. Control, 47*(2), 253-270.

[7] Zhu, X. L., & Yang, G. H. (2008). Jensen integral inequality approach to stability analysis of continuous-time systems with time-varying delay. *IET Control Theory Appl., 2*(6), 524-534.

[8] Yang, Z., & Yang, Y. P. (2010). New delay-dependent stability analysis and synthesis of T-S fuzzy systems with time-varying delay. *Int. J. Robust Nonlinear Control*, *20*(3), 313-322.

[9] Zhu, X. L., & Yang, G. H. (2010). New results of stability analysis for systems with time-varying delay. *Int. J. Robust Nonlinear Control*, *20*(5), 596-606.

[10] Chiou, J. S. (2006). Stability analysis for a class of switched large-scale time-delay systems via time-switched method. *IEE Proc.-Control Theory Appl., 153*(6), 684-688.

[11] Sun, X. M., Wang, W., Liu, G. P., & Zhao, J. (2008). Stability analysis for linear switched systems with time-varying delay. *IEEE Trans. Syst., Man, Cybern. B, Cybern., 38*(2), 528-533.

[12] Sun, Y. G., Wang, L., & Xie, G. (2009). Exponential stability of switched systems with interval time-varying delay. *IET Control Theory Appl., 3*(8), 1033-1040.

[13] Tissir, E. H. (2011). Exponential stability of switched linear systems with mixed time delays. *ICIC Express Letters, 5*(7), 2101-2109.

時變延遲互連切換系統之改良式延遲導數相關漸近穩定準則

楊明憲
建國科技大學電機工程系

摘要

本文旨在探討時變延遲互連切換中立型系統之延遲相關漸近穩定度測試問題。藉由加權延遲方法、奇異模型轉換技巧、芬斯勒補助定理、擴展型李亞普諾-克羅斯威斯基泛函數，針對上述系統，提出改良式延遲導數相關漸近穩定測試準則。本文所提之準則表示為矩陣不等式形式，可便於軟體模擬求解。舉例證實本研究方法明顯改善文獻結果。

關鍵字：互連切換中立型系統、時變延遲、加權延遲方法、奇異模型轉換、延遲導數相關穩定準則。